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Quantitative Modelling of JET Plasmas by Computational Methods

D. F. Duchs

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Quantitative modelling of JET plasmas by computational methods

By D. F. Düchs

JET Joint Undertaking, Abingdon, Oxfordshire OX14 3EA, U.K.

For quantitative predictions the tokamak plasma has to be modelled as a consistent entity.

Distinguishing features of fusion plasma theory are the simultaneous importance of a large number of effects, essential multidimensionality in geometrical and velocity space, and a high degree of nonlinearity in the interaction between these effects.

Only computational methods, mainly based on linearization, iteration and on procedures for solving huge systems of linear equations, are widely applicable, and provide quantitative 'point results' of the 'numerical experiment' type.

The practical limitations of computer capacity and cost of computing time impose severe limits on details which can be taken into account in computational plasma models.

For plasmas of tokamaks such as JET, the models are set up as initial boundary-value problems on several time scales, and composed of a cluster of interdependent computer codes.

The basic magnetic field-plasma configuration is determined from a one-fluid (magnetohydrodynamic) theory in two dimensions. The macroscopic stability of these configurations is checked. For stable plasmas the secular evolution of a sequence of equilibria is computed by 'transport codes'. In these the balance equations for the conservation quantities (mass, momentum, energy) are solved for fluxes between, and sources and sinks on, the magnetic surfaces (as determined from equilibrium). These are multifluid equations in one spatial dimension for all charged plasma species including impurities.

Important source terms such as electromagnetic wave heating, injected particles, and fusion α-particles must be calculated kinetically with at least one additional velocity-space coordinate.

At least at the plasma boundary, neutral atoms cannot be neglected. Their distribution is calculated by Monte-Carlo methods in three spatial dimensions (and velocity space). The bulk plasma is usually surrounded by magnetic surfaces or field lines that cross solid walls. Here also, for the transport of charged particles, propagation in two dimensions must be calculated.

The consistent combination of these major elements is considered.

1. Introduction

Most of the contributions to this Discussion Meeting are concerned with important *single* effects and selected plasma features. However, a tokamak plasma has also to be seen as a *whole* to put the single features in proper perspective, and to understand the interdependence and (usually nonlinear) interaction between them. It is one of the major tasks of the JET Theory Division to build up a mathematical model that describes consistently and quantitatively *all* the important aspects of the behaviour of JET plasmas with sufficient accuracy to allow extrapolation to future plasmas of tokamak type.

This paper tries to outline the basic physics assumptions from which our present plasma model has been derived and to indicate the present status of development on the way to this ambitious goal.

Where feasible, it is attempted, at least in the first few sections, to refer back to physics knowledge that is familiar to a larger audience and not only to tokamak specialists. It is, of course, not possible to present details; these can be found in books on plasma physics or traced from the references to original papers. The reference list given is, however, by no means complete and the quoted publications should serve only as typical examples.

2. Basic equations

The starting point and general scheme of our model is very traditional. As for any gas in statistical mechanics we maintain that every plasma species can be fully described by a distribution function, e.g.

$f_e(\vec{x}, \vec{w}, t)$ for electrons,

$f_j(\vec{x}, \vec{w}, t)$ for ion species j, atoms, molecules.

The coordinates in geometrical space are denoted by $\vec{x} = \{x, y, z\}$, in velocity space by $\vec{w} = \{w_x, w_y, w_z\}$, and time by t. The evolution of these functions is determined by a corresponding set of Boltzmann-type equations:

$$\frac{\partial f_k}{\partial t} + \vec{w}\frac{\partial f_k}{\partial \vec{x}} + \frac{e_k}{m_k}\left(\vec{E} + \frac{\vec{w}}{c} \times \vec{B}\right)\frac{\partial f_k}{\partial \vec{w}} = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}, \tag{0}$$

(k = any plasma species).

The right-hand side contains all collision and field fluctuation effects. Set (0) is complemented by Maxwell's equations:

$$\frac{\partial \vec{B}}{\partial t} = -c \operatorname{rot} \vec{E}, \tag{1}$$

$$\epsilon \frac{\partial \vec{E}}{\partial t} = \operatorname{rot} \vec{B} - \frac{4\pi}{c}\vec{j}, \tag{2}$$

$$\operatorname{div} \vec{B} = 0. \tag{3}$$

It is obviously impossible to solve in a straightforward way such a system for practical cases such as a JET plasma because of the lack of mathematical tools and the limitations of even the largest computers available. However, it is very useful to remember that all simplifying assumptions that lead to a practical tokamak plasma model below represent assumptions on features of the distribution functions, and relating them back to these functions helps to choose them consistently and use them as a measure for judging the degree of approximation.

3. Basic simplifying assumptions

To arrive at a practical model we use the following observations.

(*a*) The plasma is, within the chosen spatial resolution, electrically (quasi-) neutral.

(*b*) Not all plasma species are of significant importance.

(*c*) There are separable time scales: for example, many waves propagate in our plasmas very rapidly (usually in the microsecond range), whereas we are interested in the collisional (millisecond) régime.

(*d*) There are spatial symmetries, and ignorable coordinates: for example, toroidal symmetry, poloidal angle on flux surfaces.

(*e*) Transport is very much faster parallel to the magnetic field lines than perpendicular to them.

(*f*) The velocity distributions of the bulk plasma species are close to Maxwell–Boltzmann distributions.

All deviations from these basic features are taken as perturbations, and at least in first order treated by linearization or with quasilinear methods.

4. Ordering of plasma species

In table 1 particle species are collected that can all be found in a realistic JET plasma. The ideal fusion plasma is indicated by the dashed frame and consists already of eight different components (excluding H), and the heavier elements from the wall are certainly not completely negligible (JET shows at present an effective mean ion charge $Z_{eff} \approx 3$). For JET we decided to treat the species

$$e^-, H^+, D^+, T^+, \text{ and } He^{2+}$$

as *bulk* plasma species, all others as impurities with

$$Z_{imp}\, n_{imp} \ll n_e. \tag{4}$$

Table 1. Choice of 'bulk plasma' species and 'impurity' species

	ideal 'fusion plasma'					impurities		
e	H^+	D^+	T^+	He^{2+}		C^{6+}	O^{8+}	Ni^{28+}
	H^0	D^0	T^0	He^+	...	C^{5+}..	O^{7+}..	Ni^{27+}
				He^0		C^{4+}	O^{6+}	Ni^{26+}
						C^{3+}	O^{5+}	Ni^{25+}
						C^{2+}	O^{4+}	Ni^{24+}
						C^+	O^{3+}	Ni^{23+}
						C^0 ..	O^{2+}..	Ni^{22+}
							O^+	Ni^{21+}
							O^0	Ni^{20+}
								⋮
								Ni^0

If this condition (4) could not be fulfilled the plasma would hardly be suitable for a fusion reactor because of excessive losses, although the description of some present-day discharges might be doubtful with this choice.

5. Basic conservation laws

To arrive at a tractable form of (0), we take the distribution functions of the above main plasma species to be predominantly Maxwellian,

$$f_j \approx f_{Mj} + f_{1j},$$

where f_{1j} is a small correction term.

With this assumption we can approximate the distribution functions by the so-called velocity moments:

$$\left.\begin{aligned} &\text{the density} && n_j(\vec{x}, t) = \int f_j \, d^3w, \\ &\text{the flow velocity} && n_j \vec{v}_j(\vec{x}, t) = \int \vec{w}_j f_j \, d^3w, \\ &\text{the pressure tensor} && P^{(j)}_{kl}(\vec{x}, t) = m_j \int w_k w_l f_j \, d^3w, \\ &\text{and the heat flux} && Q^{(j)}_k(\vec{x}, t) = \int \tfrac{1}{2} m_j w_j^2 w_{jk} f_j \, d^3w, \end{aligned}\right\} \tag{5}$$

and higher-order moments, if desired.

Integration of the Boltzmann-type equations (0) over velocity space leads (Golant *et al.* 1980) to the familiar set of 'conservation laws' for each species j (this index has been dropped):

$$\frac{\partial n}{\partial t} + \operatorname{div}(n\vec{v}) = \int \left(\frac{\delta f}{\delta t}\right)_{\text{coll}} d^3w \quad \text{(particle sources and sinks)}, \tag{6}$$

$$mn\left\{\frac{\partial \vec{v}}{\partial t} + (\vec{v}\ \text{grad})\ \vec{v}\right\} - Zen\vec{E} - \frac{Zen}{c}(\vec{v} \times \vec{B}) + \operatorname{div} \overleftrightarrow{P})$$
$$= \int m\vec{w} \left(\frac{\delta f}{\delta t}\right)_{\text{coll}} d^3w \quad \text{(momentum sources and sinks)}, \tag{7}$$

$$\frac{\partial}{\partial t}(\tfrac{3}{2}nT) + \operatorname{div}(\tfrac{3}{2}nT\vec{v}) + nT \operatorname{div}\vec{v} + \operatorname{div}(\vec{q}) + \sum_{l,k} \pi_{lk} \frac{\partial v_e}{\partial x_k}$$
$$= \int \frac{m}{2}(w-v)^2 \left(\frac{\delta f}{\delta t}\right)_{\text{coll}} d^3w \quad \text{(energy sources and sinks)}. \tag{8}$$

It might be worthwhile to recall a few well-known features of such systems of equations that can be used to justify their choice as our basic set.

These equations are exact (for any distribution function) if the proper definitions (5) are used. So we are well advised to keep them as complete as possible.

The distribution functions are considered to be approximated well enough by the velocity moments, and those then become the basic plasma characteristics.

The system of moment equations, however, is not closed. To arrive at a closed system further assumptions on features of the distribution function must be made, mainly at the following places in (6)–(8).

(*a*) The obvious right-hand sides, which contain the collision operators. Phenomenological Ansatz are discussed below.

(*b*) Equation (8) could be a tensor equation for the components of the full pressure tensor $\overleftrightarrow{P}$ in (7). One even should expect anisotropy in a system that is strongly determined by a magnetic field. However, most fusion plasma models take as yet only the scalar part of the pressures into account and largely neglect viscosity.

(*c*) The moment expansion is truncated after the second (quadratic) moment, i.e. (8). This equation contains, however, the next higher moment, the heat flux (which would be, in general, a third-order tensor with 27 components).

The popular Ansatz

$$\vec{q} = -K \operatorname{grad}(T)$$

is certainly the most primitive one and can only be expected to describe very simple situations.

If this basic system of balance equations would turn out to be inadequate then (*a*)–(*c*) would be the most obvious starting points for improvements.

6. Plasma characteristics

Before we proceed with more assumptions to our practical model, the already reached status of simplification might be summarized for the bulk plasma:

$$\left.\begin{aligned} f_{\mathrm{e}}(\vec{x},\vec{w},t) &\rightarrow n_{\mathrm{e}}(\vec{x},t),\ \vec{v}_{\mathrm{e}}(\vec{x},t),\ T_{\mathrm{e}}(\vec{x},t),\\ f_{\mathrm{H}^+}(\vec{x},\vec{w},t) &\rightarrow n_{\mathrm{H}^+}(\vec{x},t),\ \vec{v}_{\mathrm{H}^+}(\vec{x},t),\ T_{\mathrm{H}^+}(\vec{x},t),\\ f_{\mathrm{D}^+}(\vec{x},\vec{w},t) &\rightarrow n_{\mathrm{D}^+}(\vec{x},t),\ \vec{v}_{\mathrm{D}^+}(\vec{x},t),\ T_{D^+}(\vec{x},t),\\ f_{\mathrm{T}^+}(\vec{x},\vec{w},t) &\rightarrow n_{\mathrm{T}^+}(\vec{x},t),\ \vec{v}_{\mathrm{T}^+}(\vec{x},t),\ T_{\mathrm{T}^+}(\vec{x},t),\\ f_{\mathrm{He}^{2+}}(\vec{x},\vec{w},t) &\rightarrow n_{\mathrm{He}^{2+}}(\vec{x},t),\ \vec{v}_{\mathrm{He}^{2+}}(\vec{x},t),\ T_{\mathrm{He}^{2+}}(\vec{x},t) \end{aligned}\right\}\ \text{plus small correction functions } f \quad (9)$$

A system of equations (i.e. (7)–(9)), even for such a set of fluid quantities, is not manageable if one tries to retain all three spatial coordinates.

7. Ordering of time scales

At this point we make use of the above mentioned very different time scales that can occur in fusion plasmas. Equation (7) still contains the time behaviour of all acoustic and Alfvén waves, and (2) the electromagnetic ones. We are interested, however, in the description of a whole discharge over periods of several seconds and need to consider, therefore, only the stationary forms of (7) and (2).

Adding up the resulting equations (7) for all bulk components leads to the plasma equilibrium equation

$$(1/c)\,(\vec{j}\times\vec{B}) = \operatorname{grad}\left(\sum_{\mathrm{j}} P_{\mathrm{j}}\right), \tag{10}$$

and the difference between the electron and ion equations leads to a generalized Ohm's law

$$\vec{E} = \eta\vec{j} - (1/c)\,(\vec{v}\times\vec{B}) + (1/ne)\operatorname{grad} P_{\mathrm{e}} - \alpha(\vec{B}\times\operatorname{grad} T). \tag{11}$$

The first term in this equation is often dominant. When inserted into the right-hand side of Faraday's law (1) we obtain equations for the evolution of the magnetic field.

The plasma equilibrium equation (10) is elliptic and can be solved for toroidally symmetric tokamak plasmas as the so-called Grad–Schlüter–Shafranov equation.

The resulting equilibria for axisymmetric tokamak plasmas have a very convenient feature: they lead, at least for the plasma interior, to closed and nested magnetic surfaces (see, for

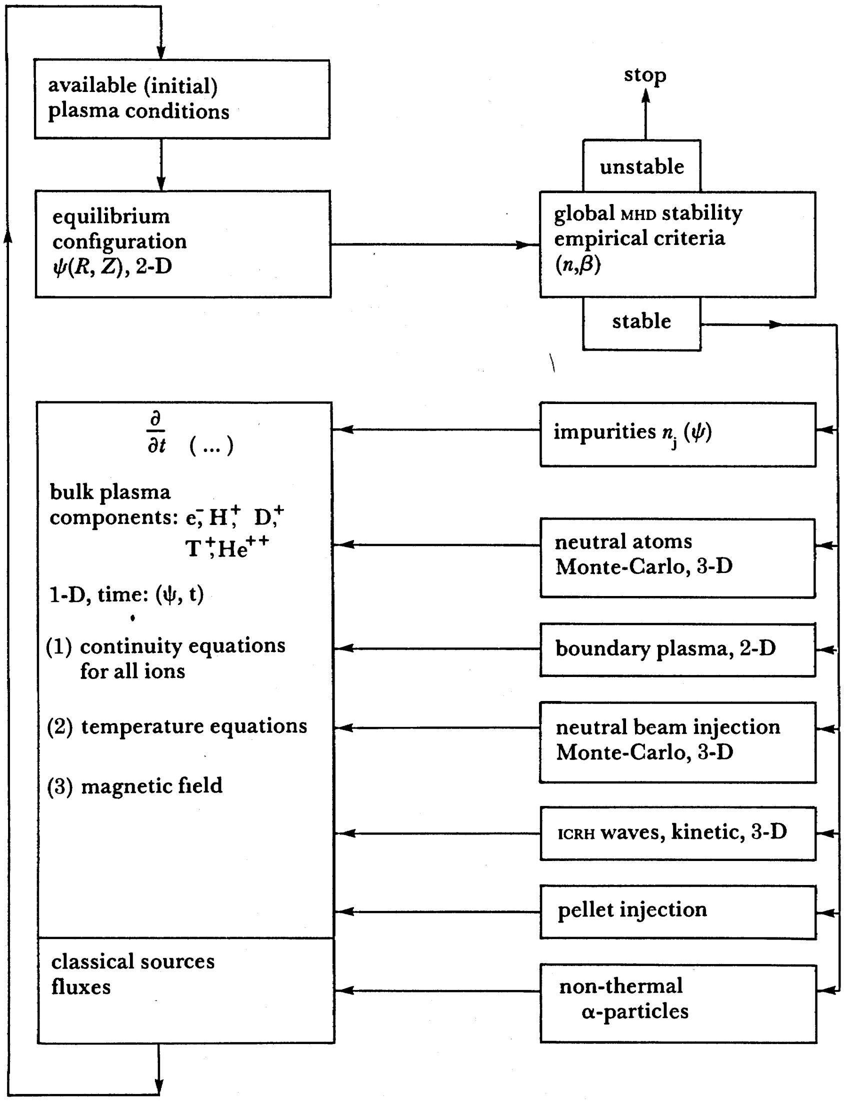

FIGURE 1. Diagram of a computational tokamak plasma model.

example, figure 2). It is well established that particle, momentum and energy transport along magnetic field lines greatly exceeds that perpendicular to $\vec{B}$. Therefore, the equilibration of plasma quantities *on* a magnetic surface can be classified as instantaneous, and transport needs to be computed only from magnetic surface to magnetic surface ('perpendicular to ψ').

This ordering finally brings the model for the bulk plasma close to practicality. The system (9) reduces to

$$\left.\begin{aligned} f_e(\vec{x},\vec{w},t) &\rightarrow n_e(\psi,t),\ T_e(\psi,t),\\ f_{H^+}(\vec{x},\vec{w},t) &\rightarrow n_{H^+}(\psi,t),\ T_{H^+}(\psi,t),\\ f_{D^+}(\vec{x},\vec{w},t) &\rightarrow n_{D^+}(\psi,t),\ T_{D^+}(\psi,t)\\ f_{T^+}(\vec{x},\vec{w},t) &\rightarrow n_{T^+}(\psi,t),\ T_{T^+}(\psi,t),\\ f_{He^{2+}}(\vec{x},\vec{w},t) &\rightarrow n_{He^{2+}}(\psi,t),\ T_{He^{2+}}(\psi,t), \end{aligned}\right\}\ \text{plus small correction functions } f \qquad (12)$$

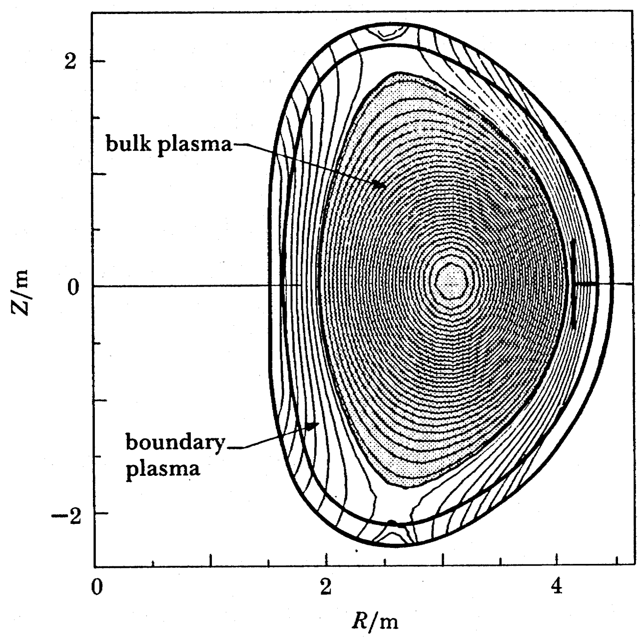

FIGURE 2. Typical cross section of magnetic flux surfaces of a tokamak equilibrium.

and the equilibrium configuration, expressed by the magnetic flux in a cross-sectional plane of the plasma torus:

$$\psi = \psi(R, Z). \tag{13}$$

8. Model schema

Before the actual status of our model is reviewed a summary as shown in figure 1 might be useful. We begin at the top of the figure and follow the arrows.

As for all initial value problems we have to start with a given plasma that might be measured or previously computed. For this given plasma we solve the equilibrium equation (10) taking into account the technical boundary conditions such as currents in coils, iron core of transformer, limiter position.

If an equilibrium exists, the set of flux surfaces determines the geometry of the problem as shown in an example in figure 2. Two regions can be distinguished, the inner one with nested closed surfaces and a boundary layer where the surfaces touch parts of the material wall. The last closed surface is considered and treated as the plasma boundary, and the 'bulk plasma' ends here.

Not all equilibria are stable; therefore, the global MHD-stability is checked together with other computationally available or empirical stability criteria. If the equilibrium turns out to be unstable the calculation is stopped.

In case of stability the conservation laws (6) and (8) are used to compute the (slow) changes in the distribution of densities, temperatures and magnetic field from ψ-surface to ψ-surface as indicated by the large box on the left-hand side of figure 1.

The main input elements in these conservation laws are fluxes (particles, energy, momentum), and source and sink terms. As pointed out above, many of these must be treated as perturbations to the bulk plasma with (sometimes considerable) deviations from the Maxwellian velocity distribution. Those phenomena which have been singled out for special treatment are shown in the right-hand side small boxes in figure 1. The procedure for their computation is rather uniform: the equilibrium geometry and the bulk plasma parameters are taken as constant (during one time step), and the particle, momentum and energy deposition (or loss) rates are computed on this constant background. These rates are then fed into the balance equations for advancing the bulk plasma parameters. If any of these rates turns out to be dominating the balance, i.e. if the time step is not chosen small enough, iteration procedures are necessary.

With respect to the deviation from the Maxwellian, the deviating part is mostly treated as a separate species which interacts with the background Maxwellian but not with itself.

Present status of model

In the following sections a few more details will be given for those model elements which are mentioned in figure 1; the present status of development will be pointed out.

9. Plasma equilibrium

With our code ESCO (Cenacchi *et al.* 1987) the two-dimensional equilibrium calculation has reached both a physically and a computationally mature state.

It can treat a free (and moving) plasma boundary and accepts any type of plasma pressure and current density profile. A finite element grid is constructed by computer for any given technical hardware configuration.

Some plasmas in JET show a non-negligible rotation, which certainly must occur with asymmetric injection of neutral beams. Such flows could affect strongly the plasma equilibria, and we are investigating the necessary modifications, i.e. studying the new equilibrium equation (Zelazny 1985).

$$mn(\vec{v}\ \mathrm{grad})\,\vec{v}+\mathrm{grad}\sum_{\mathrm{j}} P_{\mathrm{j}} = \frac{1}{c}(\vec{j}\times\vec{B}).$$

Routinely usable results are not yet available.

There seem to exist also equilibria with noticeable helical toroidal perturbations ('islands'). Such three-dimensional equilibria are not computed at present. The same methods as those used for stellarator plasmas should be applicable.

10. MHD-stability

Although MHD-instabilities form an important part of plasma behaviour they are not yet well represented in our models. As already indicated in §8 we distinguish between global and local stability.

The former refers to the very existence of any kind of stable tokamak configuration. This aspect is checked presently by empirical criteria such as β-limits, density or safety factor q-limits, or by theoretically obtained criteria (e.g. Mercier criterion).

The latter seems to be important for describing the local phenomena connected with sawteeth and other magnetic island structures. Only the sawtoothing is modelled regularly at present by a largely empirical approach that leads to enhanced transport in regions where the safety factor $q \lesssim 1$.

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11. Fluxes

As pointed out in the derivation above, all fluid type theories need special assumptions on the heat fluxes (and the particle flows if they are determined by friction as in our case).

There exists no generally accepted theory for these fluxes in fusion plasmas, and we currently use various empirical formulas.

Here lies one of the most serious uncertainties for any prediction of future plasmas. The degree of uncertainty is discussed in Bickerton (this symposium). It is planned to use, at least, the Onsager Ansatz in order to ensure some consistency, and have the formulae of the neoclassical theory to provide limiting minimum values (the maxima are, of course, much more critical).

12. Impurities

The difficulty with the impurities lies obviously in the large number of different ionic species. As for the bulk ions only the one-dimensional (ψ-dependent) evolution in time is considered. Temperatures are not computed for each species; the equilibration with the main bulk ion temperature is established rather rapidly in the usual range of plasma parameters.

The main influence of the impurities is through the radiation loss in the equation for the electron temperature T_e; especially in outer plasma layers this is often the dominant phenomenon and requires careful iteration. Other influences are as sources (or sinks) for electron density and through the increase in collisionality because of the higher ionic charge, as seen e.g. in plasma resistivity, ion heat conduction, and neutral beam absorption.

Some atomic physics input data such as ionization, recombination and excitation rates are still not known with sufficient accuracy and need continuous updating.

13. Neutral atoms

If one remembers that during one discharge the average plasma particle reaches the wall and re-enters as a neutral atom at least ten times, the importance of neutrals and recycling is obvious. Unfortunately, the neutrals are not bound to the magnetic field and must, therefore, be computed in three spatial dimensions. In addition, back scattering and long mean free paths usually create non-Maxwellian velocity distributions. However, the particle speed is so high that instantaneous spreading-out can be assumed.

For neutrals, therefore, a stationary Boltzmann equation must be solved. In our case this is accomplished by a Monte-Carlo code (Cupini *et al.* 1983).

14. Boundary plasma

As mentioned above, the bulk plasma is separated from the actual wall by a shadow plasma in open magnetic surfaces and a plasma boundary layer; these layers determine the boundary conditions for the balance equations of the bulk plasma. In these layers plasma flows and energy transport must also be computed along magnetic field lines; this introduces (at least) two spatial coordinates (Düchs 1986).

We have developed a two-dimensional transport and flow code for this region (Simonini *et al.* 1985). It is connected to the above mentioned Monte-Carlo code for neutral atoms because the neutrals provide the dominant source terms especially in this layer near the wall.

This boundary plasma computer code cannot, however, be combined with the main model for routine use. It is too expensive and too time consuming. It is instead applied for studying basic effects in detail and for bench marking simplified boundary-value models that can be run routinely with the main code.

15. Neutral injection

Injected neutral atoms provide a prime source of particles (for the injected ion species and electrons), deliver momentum to the bulk plasma, and act especially as a heat source for all bulk components. The computation can be broken down into four stages:

(*a*) formation of the neutral beam from the ion source to the plasma surface,

(*b*) absorption of the neutrals by the plasma (three-dimensional, 'optical beam' calculation),

(*c*) first orbit losses (single particle motion),

(*d*) interaction of the created fast ions with the background plasma ('slowing down').

Point (*d*) obviously requires kinetic treatment. The most elaborate code available at JET uses Monte-Carlo methods again to obtain solutions (Lister 1985).

16. Ion-cyclotron wave heating

The aim here is to compute the power density deposition from these waves into the various bulk plasma components. The computation is broken up into three major steps.

(*a*) Propagation of electromagnetic waves, launched from the antenna, in a (known) background plasma in all three dimensions, i.e. solving an equation for the electric field E

$$\partial^2 E/\partial t^2 + \nabla \times \nabla \times \vec{E} = \epsilon \vec{E}.$$

Figure 3. Distribution of the electric field produced in a cross section of a tokamak plasma irradiated by waves in the ion-cyclotron frequency range.

The dielectric tensor ϵ contains the plasma distribution function properties which are assumed to be known in this step. The equation is solved in its linearized form (for Fourier components) (Appert *et al.* 1986). An example given in figure 3 illustrates the typical complexity of the resulting E-field.

(*b*) The E-field is used as input into a Fokker–Planck equation for the resonating particle species

$$\partial f_r/\partial t = Q(E, f_r) + C(f_j)$$

with a wave absorption term Q which determines the modification of the (tail of the) distribution function averaged over a magnetic flux surface (Succi 1986). An example for such a distortion away from a Maxwellian is shown in figure 4. No spatial interdependence is taken into account at this stage.

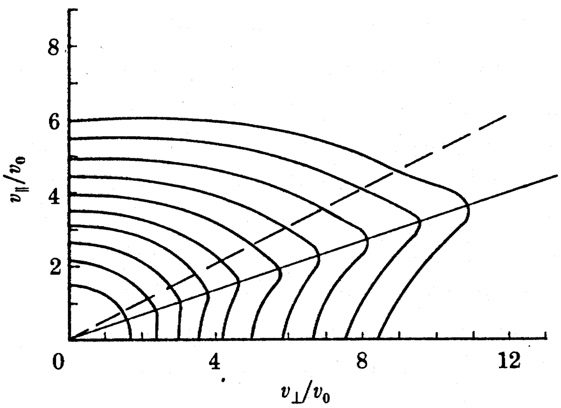

FIGURE 4. Contour lines of a velocity distribution function under the influence of ion-cyclotron radiation. Unperturbed (thermal) contour lines would be quarter circles.

(*c*) Finally, the power distribution among all plasma species is computed over collisional slowing-down similar to the case of neutral beam heating.

Again these codes (Succi 1986) are too expensive to be used at each time step. They are, at present, used to construct look-up tables for a number of JET scenarios from which very simplified power deposition profiles can be computed with reasonable accuracy.

17. Non-thermal α-particles

Fusion α-particles are born with an energy of about 3.52 MeV from which they slow down to near bulk plasma temperatures. This again obviously requires a kinetic treatment. Although it is not necessary to use it for present JET plasmas, we have a code available that computes $f_\alpha(E, \psi, t)$ from an appropriate kinetic equation (Düchs & Pfirsch 1974).

18. Pellet injection

Frozen hydrogenic pellets injected into the plasma at high speed are investigated for internal fuelling. The modelling consists of two major steps, namely calculation of

(*a*) the linear path of the pellet through the plasma torus (usually determined by inertia);

(*b*) ablation of pellet material under local plasma conditions resulting in plasma particle and neutral atom source, and temperature sink terms. These terms are then averaged over flux surfaces assuming very rapid transport parallel to $\vec{B}$.

Recent results from such modelling for JET were presented at the EPS conference in Schliersee (Watkins *et al.* 1986).

All of these 'perturbation codes' (right-hand side of figure 1) exist primarily as stand-alone codes. Already from the above very crude description it might be clear that the running of these codes is very expensive and time consuming. Thus, they are mainly used to bench mark strongly simplified models that are run routinely with the bulk plasma code. Thus, the impurities are often represented by a coronal model, the neutrals by a fluid like description, the neutral beams by a single 'pencil beam', and the MHD stability by the β_{max}-criterion.

19. Summary and conclusions

Although it is necessary (and probably more exciting) to investigate selected plasma effects in detail, any serious tokamak plasma theory needs to view the plasma as a whole for quantitative predictions. This is possible in a systematic way using concepts mainly from statistical mechanics and electrodynamics together with methods of computational physics.

For practical use many simplifications must be introduced judiciously. At present, the plasma is pictured roughly as a blend of toroidally symmetric fluids with moderate corrections for deviations, e.g. from Maxwellian velocity distributions and from geometrical symmetry.

The quantitatively important model ingredients have been identified (see figure 1). Although it is not explicitly shown here this identification is based on numerous comparisons with measurements.

The relative importance of the various ingredients changes, of course, with boundary conditions, with the evolution of a plasma in time, and for different locations inside the plasma. Certainly, none of the items is negligible for the majority of the cases of interest.

Also not proven by detailed examples, but clearly indicated is the nonlinearity of the model, which can lead to surprising interactions. Here lies much of the heuristic value of this computational work.

Finally, a remark on the success of these models: it has not been concealed in §§9–18 that there exists as yet no reliable and tractable theory for several important sub-topics such as the fluxes and local MHD stability, and that these phenomena are, at present, described by empirical formulae. Therefore, as might be expected, the computational results of our model show rather good agreement with measurements from present-day tokamaks.

Predictions beyond the empirical range, however, carry along the uncertainties of extrapolation. However, often the uncertainty factor is over-estimated because the relative weight of the 'empirical phenomena' within the full model is not properly taken into account.

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